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DJJ-003-016204 Seat No. ________

M. Sc. (Mathematics) (Sem. II) (CBCS) Examination

May / June – 2015

MATHS. CMT - 2004 : Methods in Partial Differential Equations

Faculty Code : 003

Subject Code : 016204

Time : Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1. Answer any **Seven** $7 \times 2 = 14$
   (a) Find a primitive of $ydx + xdy + 3z^2dz = 0$.
   (b) Find the integral curves of the equations $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$.
   (c) Find $curl(7y^2z^2, 5z^2x^2, 9x^2y^2)$.
   (d) Eliminate the arbitary function f from the equation $z = f(x - y)$.
   (e) Find a complete integral of $z = p^3 + q^3$.
   (f) If $F(D, D')$ is reducible and if $(2D' + 3)$ is a factor of $F(D, D')$, then verify that $z = e^{\frac{-3y}{2}}\phi(2x)$ is a solution of $F(D, D')z = 0$, where $\phi(\xi)$ is an arbitrary function of the single variable ξ.
   (g) Find a partial differential equation for which a complete integral is $2z = ay^2 + bx^2 - \frac{1}{b}$, where a and b are arbitrary constants.
   (h) Find a particular integral of $(16D'^2 - 25D^2)z = e^{2x+3y}$.
   (i) Verify that the equation $r + 2s + t = 0$ is of type parabolic.
   (j) Find a complete integral of $yp - xq = 0$.

2. Answer any **Two** $2 \times 7 = 14$
   (a) Find the integral curves of the sets of equations:
   (i) $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$
   (ii) $\frac{dx}{y(x+y)+2z} = \frac{dy}{x(x+y)-2z} = \frac{dz}{z(x+y)}$.
   (b) Find the orthogonal trajectories on the cylinder $y^2 = 2z$ of the curves in which it is cut by the system of planes $x + z = c$, where c is a parameter.
   (c) Verify that the equation $y(1 + z^2)dx - x(1 + z^2)dy + (x^2 + y^2)dz = 0$ is integrable and find its primitive.

3. (a) Find the general integral of the linear partial differential equation $(x + z)p + (y + z)q + z = 0$. 5
   (b) Find the equation of the integral surface of the partial differential equation $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0, x^2 + y^2 = 2x$. 5
   (c) (i) Find the envelope of the one-parameter system of surfaces $x^2 + y^2 + (z - a)^2 = 1$.
   (ii) Determine the envelope of the two-parameter system of surfaces $(x - a)^2 + (y - b)^2 + z^2 = 1$. 4

OR

3. (*a*) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2,\ z = 0$. 5

(*b*) Find a complete integral of the partial differential equation $xpq + yq^2 = 1$. 5

(*c*) Show that an equation of the form $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = f(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z})$ is always soluble by Jacobi's method. 4

4. Answer any **Two** $2 \times 7 = 14$

(*a*) Reduce the equation $r - x^2t = 0$ to canonical form.

(*b*) Verify that the equation $yz(y + z)dx + zx(z + x)dy + xy(x + y)dz = 0$ is integrable and find its solution.

(*c*) Prove that $yz(z^2+yz-2y) = x^2$ is a solution of $2x(y+z^2)p+y(2y+z^2)q = z^3$.

5. Answer any **Two** $2 \times 7 = 14$

(*a*) Suppose that the equation $Pdx + Qdy + Rdz = 0$ is integrable. Prove that the dot product of $X = (P, Q, R)$ and $curl X$ is equal to 0.

(*b*) Solve the equation $D^3 - 2D^2D' - DD'^2 + 2D'^3 = e^{x+y}$.

(*c*) Find a complete integral of $(p^2 + q^2)y = qz$.

(*d*) Let $F(D, D') = \sum_r \sum_s c_{rs}D^rD'^s$, where c_{rs} are constants. Prove that $F(D, D')(e^{ax+by}\phi(x, y)) = e^{ax+by}F(D + a, D' + b)\phi(x, y)$.